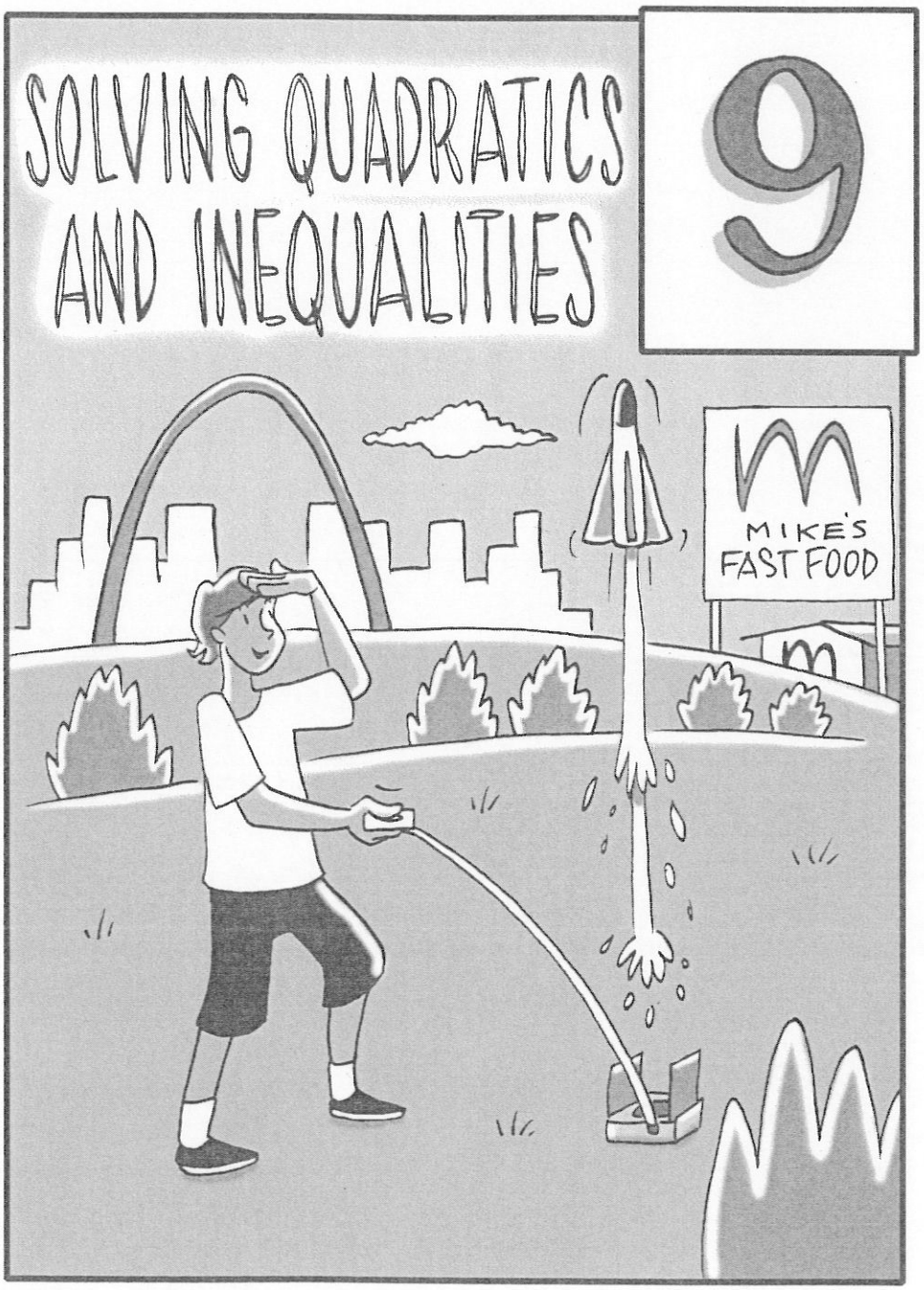
Core Connections Algebra 1



CHAPTER 9 Solving Quadratics and Inequalities

You will start this chapter by extending your ability to solve quadratic equations, and deciding which method of solving is most efficient.

So far in this course you have focused on what you can determine when two expressions are equal. By using what you know about balancing equations, you can now solve linear and quadratic equations for a given variable.

However, what if the two expressions are not equal? If you know that one expression is always larger than the other, what does that tell you about the variable? In this chapter you will learn how to deal with these types of relationships, called *inequalities*. You will develop ways to represent solutions to inequalities both algebraically and graphically.

Guiding Question

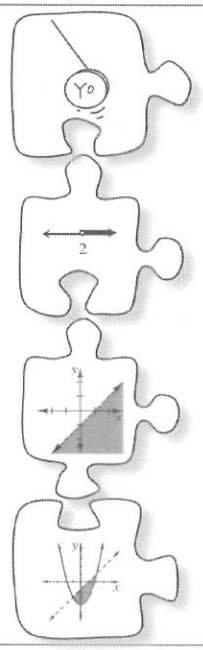
Mathematically proficient students look for and make use of structure.

As you work through this chapter, ask yourself:

Can I look closely to see a pattern or structure in these functions?

In addition, you will extend your ability to work with mathematical sentences by learning how to write inequalities that describe situations.

Chapter Outline



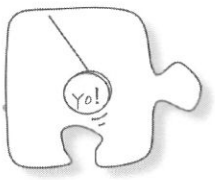
Section 9.1 In this section, you will solve quadratic equations using the Quadratic Formula.

Section 9.2 In this section, you will study how to solve linear inequalities and apply this understanding to solving applications.

Section 9.3 After learning how to represent solutions to one-variable inequalities on a number line, you will study how to represent the solutions of two-variable inequalities on an $x \rightarrow y$ graph.

Section 9.4 In the final section, you will apply what you know about systems of equations to help find the solutions to a system of inequalities.

9.1.1 What else can I solve?



Solving Quadratic Equations

In Chapter 8, you developed a method for finding the x-intercepts of a parabola given by $y = ax^2 + bx + c$. You found the roots or zeros of the quadratic expression by setting it to zero: $ax^2 + bx + c = 0$. Today you will learn how to use that skill to solve a wide variety of quadratic equations.

9-1. Use the Zero Product Property to solve the quadratic equations below.

a. $9 = 3x^2 + 4x - 6$ b. $0 = 3(x-5)(2x+3)$ c. $x^2 + 6x = 0$

9-2. Let's use your calculator to verify your solutions to the original equations in problem 9-1 (before you started rewriting them).

a. Graph $y = 3x^2 + 4x - 6$ with your graphing calculator. Where on the graph would you look to verify your solution to part (a) of problem 9-1?

b. Where in the table on your graphing calculator would you look to verify your solution to part (a) of problem 9-1?

c. Use your graphing calculator to verify your solutions to the other equations in problem 9-1.

9-3. Janelle's team was trying to solve $x^2 - 6x - 1 = 0$.

Janelle suggested using the Zero Product Property.

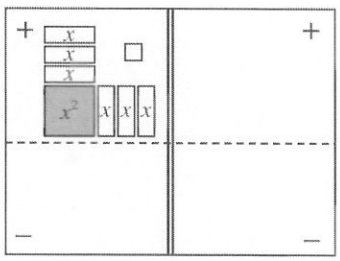
Kira said, *"This quadratic is not factorable. So it doesn't have a solution!"*

a. Is Kira correct? How do you know?

"I know. Let's use the graphing calculator and see if there are solutions." Katelyn said.

b. What did Katelyn mean? If you have not already done so, estimate the solution.

"But," Janelle said, *"the teacher said to get an exact solution. Let's complete the square to help us solve. Here's what the equation would look like."*



$x^2 - 6x - 1$

Erika contributed, *"Oh, I see. Now we can add –8 unit tiles to each side to complete the square."*

"I'm not so sure," said Katelyn. *"That would give us –9 in the corner. Is that right?"*

c. What do you think the teams next legal tile move should be?

d. Complete the square, and solve. Does your solution match your estimate from part (b)?

e. To make sense of the x-intercepts in part (d), you probably wrote your solution as a decimal. Square roots (that are not perfect squares) are **irrational numbers**: they are numbers that cannot be written as a fraction of integers. Irrational numbers cannot be written exactly as a decimal. The decimals continue infinitely and never go into a repeating pattern.

When you write a square root as a decimal you have to round, so it is called **approximate decimal form**. When you use the square root symbol, you are using **exact form** or **radical form**.

Build the equation $x^2 + 4x - 5 = -2$ with algebra tiles. Use Kira's method to complete the square and solve. Write your answer in both exact form and approximate decimal form.

f. Check your answer to part (e) by graphing.

9-4. Solve the following equations using Kira's method. Use generic rectangles to help you complete the square.

a. $x^2 - 2x - 3 = 1$

b. $8x^2 + 40x = 238$

9-5. Use your graphing calculator to verify your solutions to the original equations in problem 9-4.

METHODS AND MEANINGS

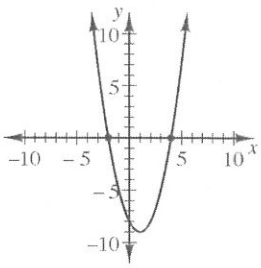
Zeros and Roots of Quadratics

MATH NOTES

A **root** or **zero** of a quadratic expression is a value of x that makes the expression equal to zero. For example, the roots or zeros of the quadratic expression $x^2 - 2x - 8$ are the solutions to the equation $x^2 - 2x - 8 = 0$.

The x-intercepts of any quadratic function are roots. You find the x-intercepts by setting the function equal to zero and solving for x.

For example, the quadratic function $f(x) = x^2 - 2x - 8 = (x+2)(x-4)$ is graphed at right. The x-intercepts are at $(0, -2)$ and $(0, 4)$. The roots or zeros are -2 and 4 because the solutions to the equation $x^2 - 2x - 8 = 0$ are -2 and 4.



Review & Preview

9-6. Use your generalized process of completing the square to rewrite and solve each quadratic equation below.

a. $w^2 + 28w + 52 = 0$

b. $x^2 + 5x + 4 = 0$

c. $k^2 - 16k - 17 = 0$

d. $z^2 - 1000z + 60775 = 0$

9-7. For each of the following equations, indicate whether its graph would be a line or a parabola.

a. $5x + 2y = 7$

b. $y = 3x^2$

c. $y = 3$

d. $4x^2 + 3x = 7 + y$

9-8. **Multiple Choice:** Which equations below are equivalent to:

$$\tfrac{1}{2}(6x - 14) + 5x = 2 - 3x + 8\ ?$$

a. $3x - 7 + 5x = 10 - 3x$

b. $3x - 14 + 5x = 2 - 3x + 8$

c. $8x - 14 = 10 - 3x$

d. $6x - 14 + 10x = 4 - 6x + 16$

9-9. Rewrite each radical expression in exponent form.

a. $\sqrt[3]{10}$

b. $\sqrt{15}$

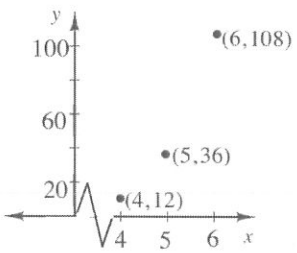
c. $\sqrt[4]{18}^{\,3}$

d. $\frac{1}{\sqrt{5}}$

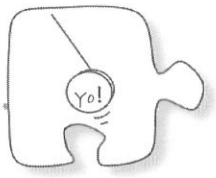
9-10. Examine the two equations below. Where do they intersect?

$$y = 4x - 3$$
$$y = 9x - 13$$

9-11. Find an equation of the sequence represented by this graph.



9.1.2 What if it is not factorable?



Introduction to the Quadratic Formula

In the previous lesson you developed methods to solve quadratic equations. Today you will learn a new method to solve them.

9-12. A quadratic equation can be rewritten in **perfect square form**, $(ax+b)^2 = c$, by completing the square.

Write the following equations in perfect square form. Then determine the number of solutions for each quadratic equation. You do not need to actually solve the equations.

Explain how you can quickly determine how many solutions a quadratic equation has once it is written in perfect square form.

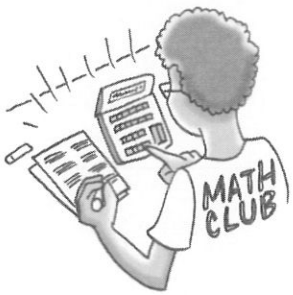
a. $x^2 - 6x + 10 = 3$

b. $m^2 + 12m + 37 = 0$

c. $4p^2 + 36p = -81$

d. $k^2 - 3k + 9 = 0$

9-13. QUADRATIC FORMULA



One way to calculate the roots of a quadratic equation is by using the **Quadratic Formula**, shown below. This formula uses values a, b, and c from a quadratic equation written in standard form (explained in the next paragraph).

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a. When the quadratic equation is written in **standard form** (i.e., it looks like $ax^2 + bx + c = 0$), then a is the number of x^2 terms, b is the number of x-terms, and c is the constant. If $x^2 - 3x - 7 = 0$, then what are a, b, and c?

Problem continues on next page. →

9-13. *Problem continued from previous page.*

b. The Quadratic Formula calculates *two* possible answers by using the "$\pm$" symbol. This symbol (read as "plus or minus") is shorthand notation that tells you to calculate the formula twice: once with addition and once with subtraction in the numerator. Therefore, every Quadratic Formula problem is really two different problems unless the value of $\sqrt{b^2-4ac}$ is 0.

$$\pm$$

$$x=\frac{-b+\sqrt{b^2-4ac}}{2a} \quad \text{or} \quad x=\frac{-b-\sqrt{b^2-4ac}}{2a}$$

Carefully substitute a, b, and c from $x^2-3x-7=0$ into the Quadratic Formula. Simplify each expression (once using addition and once using subtraction) to solve for x. Write your solution for x in both exact form and approximate form.

c. Make a graph with your graphing calculator to verify your solution.

9-14. The Quadratic Formula is only one of the tools you can use to solve quadratic equations.

a. What are the other methods that you can use?

b. You may be thinking, "Where did this formula come from? Why does it work?" You can find the formula by starting with a generic quadratic $ax^2+bx+c=0$ and using your algebra skills to solve for x. See the Math Notes box for this lesson to learn about one way this formula can be derived. Later, in Chapter 10, you will learn another formal method to derive the Quadratic Formula.

9-15. Use the Quadratic Formula to solve the equations below for x, if possible. Check your solutions with your graphing calculator.

a. $3x^2+7x+2=0$

b. $2x^2-9x-35=0$

c. $8x^2+10x+3=0$

d. $x^2-5x+9=0$

9-16. In your Learning Log, describe how to use the Quadratic Formula. Be sure to include an example. Title this entry "Quadratic Formula" and include today's date.

MATH NOTES

METHODS AND MEANINGS

The Quadratic Formula

Why is $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ a solution of $ax^2+bx+c=0$? One way to derive this formula is shown below.

1. Begin with the quadratic equation in standard form.

$$ax^2+bx+c=0$$

2. Multiply each side by $4a$.

$$4a(ax^2+bx+c)=4a(0)$$
$$4a^2x^2+4abx+4ac=0$$

3. Add b^2-4ac to each side in order to get a factorable quadratic on the left.

$$4a^2x^2+4abx+b^2=b^2-4ac$$

4. The left side can be factored as $(2ax+b)^2$, which is demonstrated in the generic rectangle shown at right.

$$(2ax+b)^2=b^2-4ac$$

b	$2abx$	b^2
$2ax$	$4a^2x^2$	$2abx$
	$2ax$	b

5. Take the square root of each side. Since a square root refers to the *positive* root, the absolute value of $2ax+b$ is used. Then by "looking inside" there are two possible values for $2ax+b$: $+\sqrt{b^2-4ac}$ and $-\sqrt{b^2-4ac}$.

$$|2ax+b|=\sqrt{b^2-4ac}$$

6. Now continue to solve for x by subtracting b from both sides and dividing by $2a$. Notice that a cannot equal zero or else you will get an error! However, if $a=0$, then this equation would not be quadratic and you would not use this formula.

$$2ax+b=\pm\sqrt{b^2-4ac}$$
$$2ax=-b\pm\sqrt{b^2-4ac}$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Thus, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ are solutions of the equation $ax^2+bx+c=0$.

Review & Preview

9-17. Solve the following quadratic equations by factoring and using the Zero Product Property. Be sure to check your solutions.

a. $x^2-13x+42=0$

b. $0=3x^2+10x-8$

c. $2x^2-10x=0$

d. $4x^2+8x-60=0$

9-18. Use the Quadratic Formula to solve $x^2 - 13x + 42 = 0$. Did your solution match the solution from part (a) of problem 9-17?

9-19. Does a quadratic equation always have two solutions? That is, does a parabola always intersect the x-axis twice?

a. If possible, draw an example of a parabola that only intersects the x-axis once.

b. What does it mean if the quadratic equation has no solution? Draw a possible parabola that would cause this to happen.

9-20. Find the equation of the line through the point $(-2, 8)$ with slope $\frac{1}{2}$.

9-21. For each quadratic function below, use the idea of completing the square to write it in graphing form. Then state the vertex of each parabola.

a. $f(x) = x^2 + 4x + 5$

b. $f(x) = x^2 - 6x$

c. In both graphs, does the vertex represent the maximum or minimum value of the parabola?

9-22. The increased demand for vegetarian meals has caused an increase in the price of tofu. If the current cost of \$2.99 per pound is increasing 6% per year, what will it cost in 5 years?

9-23. Decide if the statements below are true or false. If necessary, review the descriptions for the inequality symbols $<$, $\leq$, $>$, and $\geq$ in the Lesson 9.2.1 Math Notes box.

a. $11 < -13$

b. $5(2) \geq 10$

c. $13 > -3(2-6)$

d. $4 \leq 4$

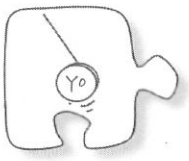
e. $9 \geq -9$

f. $-2 > -2$

g. $-16 < -15$

h. $0 > 6$

9.1.3 What if the equation is not in standard form?



More Solving Quadratic Equations

Today you will apply and extend what you know about solving quadratic equations.

9-24. For the quadratic equation $6x^2+11x-10=0$:

a. Solve it using the Zero Product Property.

b. Solve it using the Quadratic Formula.

c. Did the solutions from parts (a) and (b) match? If not, why not?

9-25. As the Math Notes box from Lesson 9.1.2 demonstrated, the Quadratic Formula can solve any quadratic equation in the form $ax^2+bx+c=0$ if $a \neq 0$. But what if the equation is not in standard form? What if terms are missing? Consider these questions as you solve the quadratic equations below. Share your ideas with your teammates and be prepared to demonstrate your process for the class.

a. $4x^2-121=0$

b. $2x^2-2-3x=0$

c. $15x^2-165x=-630$

d. $36x^2+25=60x$

9-26. THE SAINT LOUIS GATEWAY ARCH

Arches, like the Saint Louis Gateway Arch (pictured at right) are not parabolas. But sometimes arches can be *modeled* by a parabola so that predictions are possible. Suppose the Gateway Arch can be modeled by $y=630-0.00635x^2$, where both x and y represent distances in feet and the origin is the point on the ground directly below the arch's apex (its highest point).

a. Find the x-intercepts of the Gateway Arch. What does this information tell you? Use a calculator to evaluate your answers.

b. How wide is the arch at its base?

c. How tall is the arch? How did you find your solution?

d. Draw a quick sketch of the arch on graph paper, labeling the axes with all of the values you know.

MATH NOTES

METHODS AND MEANINGS

Solving a Quadratic Equation

So far in this course, you have learned three algebraic methods to solve a quadratic equation of the form $ax^2+bx+c=0$.

Example 1: Solve $3x^2+x-14=0$ for x using the Zero Product Property.

Solution: First, factor the quadratic so it is written as a product: $(3x+7)(x-2)=0$. (If factoring is not possible, one of the other methods of solving must be used.) The Zero Product Property states that if the product of two terms is 0, then at least one of the factors must be 0. Thus, $3x+7=0$ or $x-2=0$. Solving these equations for x reveals that $x=-\frac{7}{3}$ or that $x=2$.

Example 2: Solve $3x^2+x-14=0$ for x using the Quadratic Formula.

Solution: This method works for *any* quadratic. First, identify a, b, and c. a equals the number of x^2 terms, b equals the number of x terms, and c equals the constant. For $3x^2+x-14=0$, $a=3$, $b=1$, and $c=-14$. Substitute the values of a, b, and c into the Quadratic Formula and evaluate the expression twice: once with addition and once with subtraction. Examine this method below:

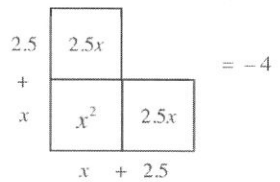
$$x=\frac{-1+\sqrt{1^2-4(3)(-14)}}{2\cdot 3} \qquad x=\frac{-1-\sqrt{1^2-4(3)(-14)}}{2\cdot 3}$$

$$=\frac{-1+\sqrt{169}}{6} \qquad \text{or} \qquad =\frac{-1-\sqrt{169}}{6}$$

$$=\frac{12}{6}=2 \qquad =\frac{-14}{6}=-\frac{7}{3}$$

Example 3: Solve $x^2+5x+4=0$ by completing the square.

Solution: This method works most efficiently when the coefficient of x^2 is 1. Rewrite the equation as $x^2+5x=-4$. Rewrite the left side as an incomplete square:



Complete the square and rewrite as $(x+2.5)^2-6.25=-4$ or $(x+2.5)^2=2.25$

Take the square root of both sides, $x+2.5=\pm 1.5$. Solving for x reveals that $x=-1$ or $x=-4$.

Review & Preview

9-27. Solve the following quadratic equations by factoring and using the Zero Product Property. Then check your solutions.

a. $x^2 - 10x + 25 = 0$

b. $0 = 3x^2 + 17x - 6$

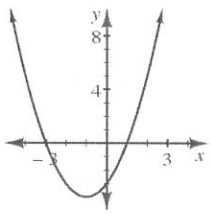
c. $3x^2 - 2x = 5$

d. $16x^2 - 9 = 0$

9-28. Use the Quadratic Formula to solve part (b) of problem 9-27 above. Did your solution match the solution you got by factoring and using the Zero Product Property (in part (b) of problem 9-27)?

9-29. Find the equation that represents the information given below.

a.



b.

x	−4	−3	−2	−1	0	1	2	3	4
y	12	5	0	−3	−4	−3	0	5	12

9-30. Solve the following problem using any method. Write your solution as a sentence.

The length of a rectangle is 5 cm longer than twice the length of the width. If the area of the rectangle is 403 square centimeters, what is the width?

9-31. Which of the points below is a solution to $4x-3y=10$? Note: More than one point may make this equation true.

a. $(1, 2)$ b. $(4, 2)$ c. $(7, 6)$ d. $(4, -3)$

9-32. In order to quickly get people between terminals in the Minneapolis Airport, long "people mover" conveyor belts were installed. Assume that if someone stood still on a conveyor belt, that person would travel 2 feet per second.

a. Since Jung is in a hurry, he decided to walk on the conveyor belt (in the same direction he would travel standing still). If his terminal is 300 feet away and he wants to get there in 60 seconds, how fast does he need to walk with respect to the conveyor belt? (Assume he can ride the conveyor belt the entire distance.)

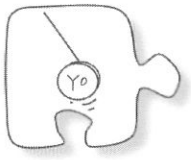
b. Jacob, who is four years old, decided it would be fun to walk on the conveyor belt in the "wrong" direction (i.e., in the direction opposite to which he would travel if standing still). If he walks for 18 seconds at a rate of 1 foot per second, how far will he travel? In what direction does he travel? Explain.

9-33. Factor each polynomial completely.

a. $3x^3-3x^2$ b. $2x^2-10x+12$

c. $8x^2-32$ d. $4x^3+10x^2-24x$

9.1.4 Which method should I use?



Choosing a Strategy

You now have more than one algebraic method to solve quadratic equations: using the Zero Product Property, using the Quadratic Formula, and completing the square. How can you decide which strategy is best to try first? By the end of this lesson, you should have some strategies to help you determine which method to try first when solving a quadratic equation.

9-34. Examine the quadratic equations below with your team. For each equation:

- Decide which strategy is best to try first.
- Solve the equation. If your first strategy does not work, switch to another strategy.
- Check your solution(s).

Be prepared to share your process with the class.

a. $x^2+12x+27=0$

b. $0.5x^2+9x-2.8=6$

c. $(3x+4)(2x-1)=0$

d. $x^2+12=8x-4$

e. $x^2+5-2x=0$

f. $20x^2-30x=2x+45$

9-35. LEARNING LOG

With the class, decide when it is best to solve a quadratic by factoring or completing the square, and when you should go directly to the Quadratic Formula. Copy your observations in your Learning Log. Title this entry "Choosing a Strategy to Solve Quadratics" and include today's date.

9-36. While solving $(x-5)(x+2)=-6$, Kyle decided that x must equal 5 or -2. "Not so fast!" exclaimed Stanton, "The product does not equal zero. We need to change the equation first."

a. What is Stanton talking about?

b. How can the equation be rewritten? Discuss this with your team and use your algebraic tools to rewrite the equation so that it can be solved.

c. Solve the resulting equation from part (b) for x.

d. Use the graph and table of $y=(x-5)(x+2)$ on your graphing calculator to check your solution. See whether Kyle's solution matches the solution in part (c).

9-37. MOE'S YO

Moe is playing with a yo-yo. He throws the yo-yo down and then pulls it back up. The motion of the yo-yo is represented by the equation $y=2x^2-4.8x$, where x represents the number of seconds since the yo-yo left Moe's hand, and y represents the vertical height in inches of the yo-yo with respect to Moe's hand. Note that when the yo-yo is in Moe's hand, $y=0$, and when the yo-yo is below his hand, y is negative.

a. How long is Moe's yo-yo in the air before it comes back to Moe's hand? Write and solve a quadratic equation to find the times that the yo-yo is in Moe's hand.

b. How long does it take for the yo-yo to turn around, that is, to start its return to his hand? Use what you know about parabolas to help you.

c. How long is the yo-yo's string? That is, what is y when the yo-yo changes direction?

d. Draw a sketch of the graph representing the motion of Moe's yo-yo. On the sketch, label the important points: when the yo-yo is in Moe's hand and when it changes direction.

METHODS AND MEANINGS

Simplifying Square Roots

MATH NOTES

Before calculators were universally available, people who wanted to use approximate decimal values for numbers like $\sqrt{45}$ had a few options:

1. Carry around copies of long square-root tables.
2. Use "guess and check" repeatedly to get desired accuracy.
3. "Simplify" the square roots. A square root is simplified when there are no more perfect square factors (square numbers such as 4, 25, and 81) under the radical sign.

Simplifying square roots was by far the fastest method. People factored the number as the product of integers hoping to find at least one perfect square number. They memorized approximations of the square roots of the integers from one to ten. Then they could figure out the decimal value by multiplying these memorized facts with the roots of the square numbers. Here are some examples of this method.

Example 1: Simplify $\sqrt{45}$.

First rewrite $\sqrt{45}$ in an equivalent factored form so that one of the factors is a perfect square. Simplify the square root of the perfect square. Verify with your calculator that both $3\sqrt{5}$ and $\sqrt{45} \approx 6.71$.

Example 1

$$\begin{aligned}\sqrt{45} &= \sqrt{9 \cdot 5}\\ &= \sqrt{9} \cdot \sqrt{5}\\ &= 3\sqrt{5}\end{aligned}$$

Examine **Examples 2 and 3** at right. Note that in Example 3, $\sqrt{72}$ was rewritten as $\sqrt{36} \cdot \sqrt{2}$, rather than as $\sqrt{9} \cdot \sqrt{8}$ or $\sqrt{4} \cdot \sqrt{18}$, because 36 is the largest perfect square factor of 72. However, since

Example 2

$$\begin{aligned}&\sqrt{27}\\ &= \sqrt{9} \cdot \sqrt{3}\\ &= 3\sqrt{3}\end{aligned}$$

Example 3

$$\begin{aligned}&\sqrt{72}\\ &= \sqrt{36} \cdot \sqrt{2}\\ &= 6\sqrt{2}\end{aligned}$$

$$\sqrt{4} \cdot \sqrt{18} = 2\sqrt{9 \cdot 2} = 2\sqrt{9} \cdot \sqrt{2} = 2 \cdot 3\sqrt{2} = 6\sqrt{2} \quad \text{and}$$
$$\sqrt{9} \cdot \sqrt{8} = 3\sqrt{4 \cdot 2} = 3\sqrt{4} \cdot \sqrt{2} = 3 \cdot 2\sqrt{2} = 6\sqrt{2},$$

you can still get the same answer if you simplify it using different methods.

When you take the square root of an integer that is not a perfect square, the result is a decimal that never repeats itself and never ends. It is a number that cannot be written as a fraction using integers. This result is called an **irrational number**. The irrational numbers and the rational numbers together form the **real numbers**.

Generally, since it is now the age of technology, when a **decimal approximation** of an irrational square root is desired, a calculator is used. However for an exact answer, called **exact form** or **radical form**, the number must be written using the $\sqrt{\ }$ symbol.

Review & Preview

9-38. Write and solve an equation (or system of equations) for the situation described below. Define your variable(s) and write your solution as a sentence.

Daria has 18 coins that are all nickels and quarters. The number of nickels is 3 more than twice the number of quarters. If she has \$1.90 in all, how many nickels does Daria have?

9-39. Solve the following quadratic equations using any method.

a. $10000x^2 - 64 = 0$

b. $9x^2 - 8 = -34x$

c. $2x^2 - 4x + 7 = 0$

d. $3.2x + 0.2x^2 - 5 = 0$

9-40. Find a rule that represents the number of tiles in Figure x for the tile pattern at right.

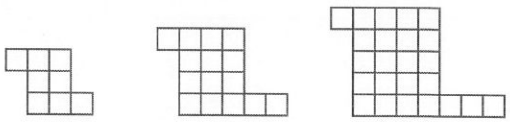


Figure 1 Figure 2 Figure 3

9-41. Solve the equations below for x. Check your solutions.

a. $3x^2 + 3x = 6 + 3x^2$

b. $\frac{5}{x} = \frac{1}{3}$

c. $5 - (2x - 3) = -3x + 6$

d. $6(x - 3) + 2x = 4(2x + 1) - 22$

9-42. Line L passes through the points $(-44, 42)$ and $(-31, 94)$, while line M has the rule $y = 6 + 3x$. Which line is steeper? Justify your answer.

9-43. No Payments For The First Six Months!

You just bought a new tablet computer for \$500 including tax. There are no required payments for six months but the company does charge 30% annual interest, compounded monthly, on any unpaid balance.

a. What is the monthly multiplier?

b. If you do not make any payments for six months, how much will you now owe for your new tablet?

9.2.1 What if the quantities are not equal?

Solving Linear, One-Variable Inequalities

In this course, you have developed a variety of skills to find solutions to different kinds of equations. Now you will apply these equation-solving skills to solve inequalities.

9-44. As a class, create a "human number line" for each of the following mathematical sentences. You will be assigned a number to represent on the number line. When your number makes the equation or inequality true, stand up to show that your number is a solution. If your number does not make the equation or inequality true, remain seated.

a. $x \geq -2$ b. $x \leq 1$ c. $x = 3$ d. $x \geq 0$

e. $x = -2$ f. $-1 \leq x \leq 4$ g. $x^2 \geq 4$ h. $x < -3$

9-45. Based on your observations from problem 9-44, discuss the following questions with your class. Be sure to justify your responses.

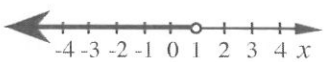
a. Compare the solutions to an inequality (like $x \geq -2$) with that of an equation (like $x = 3$). What is different? What causes this to happen?

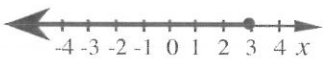
b. How many solutions does an inequality such as $x \leq 1$ have?

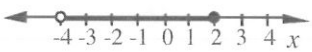
c. How is the result of $-1 \leq x \leq 4$ different from the other inequalities? What about the result of $x^2 \geq 4$?

9-46. Write an inequality that represents the solutions shown on each number line below.

a.

b.


c.


d.


9-47. SOLUTIONS TO A LINEAR INEQUALITY

With your study team, find at least five x-values that make the inequality below true:

$$2x - 5 \geq 3$$

a. How many solutions are there?

b. What is the smallest solution for x? This point is called a **boundary point**.

c. What is the significance of the boundary point? What is its relationship with the inequality $2x - 5 \geq 3$?

d. Write an inequality that represents the solutions for x. On a number line, highlight the solutions for x. Be ready to share your number line with the class.

9-48. SOLVING LINEAR INEQUALITIES WITH ONE VARIABLE

Analyze the process for solving an inequality, such as $3 - 2x < 1$, by addressing the questions below.

a. The key point to start with is the boundary point. How can you quickly solve for this point? Once you have determined your strategy, find the boundary point for $3 - 2x < 1$.

b. Decide if the boundary point is part of the solution to the inequality. If it *is* part of the solution, indicate this on a number line with a filled circle (point). If it is *not* a solution, show this by using an open circle as a boundary point.

c. Finally, to determine on which side of the boundary the solutions lie, choose a point to test in the inequality. If the point *is* a solution, then all points on that side of the boundary are part of the solution. If the point is *not* a solution, what does that tell you about the solutions? Write your solutions to $3 - 2x < 1$ as an inequality and represent the solutions on a number line.

9-49. With your study team, find all of the solutions to the inequality $3x + 1 < 7$. Decide how to represent these solutions on a number line and be prepared to justify your decisions to the class.

METHODS AND MEANINGS

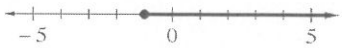
MATH NOTES

Inequality Symbols

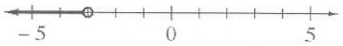
Just as the symbol "=" is used to represent that two quantities are equal in mathematics, the **inequality symbols** at right are used when describing the relationships between quantities that are not necessarily equal.

$<$ less than
$\leq$ less than or equal to
$>$ greater than
$\geq$ greater than or equal

When graphing an inequality on a number line, such as $x \geq -1$, a filled circle (point) indicates that the value is a solution of the inequality, as shown at right.

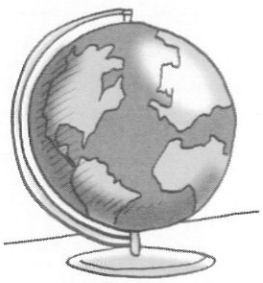


An open circle indicates that the value is not part of the solution, as in $x < -3$, as shown at right.



Review & Preview

9-50. Solve the problem below by writing and solving an equation. Be sure to define your variable.



There are a total of 122 countries in Africa, Europe, and North America (as of 2012). Europe has twice as many countries as North America, and Africa has seven more than Europe. How many countries are in each of these three continents? Write an equation and solve it to answer this question.

9-51. Solve each of the following inequalities for the given variable. Represent your solutions on a number line.

a. $2(3p+1) > -4$ b. $9k - 2 < 3k + 10$ c. $5 - h \geq 4$

9-52. Solve the following quadratic equations. Check your solutions, if possible.

a. $2k^2 + k - 6 = 0$

b. $m^2 = 9$

c. $w(2w + 8) = 24$

d. $3n^2 - 4n = 5$

9-53. Identify the statements below as sometimes true, always true, or never true.

a. $-4 \le 9$

b. $x < 1$

c. $-5 > -2$

d. $3x + 5 = 2$

e. $61 = 61$

f. $-6 < -6$

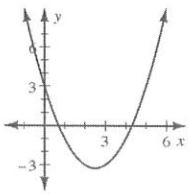
9-54. Robbie builds model rockets. One day he sets up a rocket, backs away from the launch pad, and then shoots the rocket off into the air. The rocket's path is represented by the equation $y = -10x^2 + 130x - 400$, where y is the height in meters off the ground and x is the horizontal distance in meters from Robbie.

a. Use either the Zero Product Property or the Quadratic Formula to find the x-intercepts of the path of Robbie's rocket. What do the x-intercepts tell you?

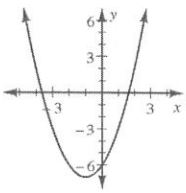
b. When Robbie's rocket lands, how far is it from the launch pad?

9-55. For each parabola graphed below, visually estimate the x-intercepts. Then use the Quadratic Formula to confirm your estimates.

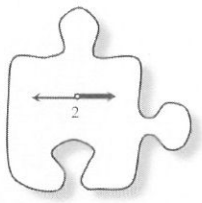
a. $y = x^2 - 5x + 3$



b. $y = x^2 + 2x - 6$



9.2.2 How can I use inequalities?



More Solving Inequalities

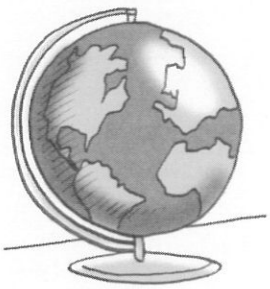
In Lesson 9.2.1 you learned how to solve inequalities with one variable. Today you will focus on special inequalities and learn how you can use inequalities to solve an application problem.

9-56. Review what you learned in Lesson 9.2.1 to solve the inequalities below. Represent your solutions both as an inequality and on a number line.

a. $x-7<-2$ b. $3m+2\leq 8m-8$ c. $\frac{2}{3}p-2>-4$

d. $2-3(x-1)\geq x-7$ e. $9k-4+1\leq 2k-3+7k$ f. $3y+1<3y+1$

9-57. THE UNITED NATIONS



At the end of this chapter, your team will have the exciting responsibility of representing a country at a special meeting of the United Nations (U.N.). The U.N. needs your help preparing for future large-scale disasters. You will need to help find a solution that not only works best for the country you represent, but that also accommodates the needs of each of the other countries. To prepare you for this task, the next several lessons will present daily problems to familiarize you with the important issues and concerns of other countries.

Start by writing and solving an equation (or system of equations) that represents the problem below. Be sure to define any variables you use.

Turkey has a population of 66 million people and is made up almost entirely of two ethnic groups: Turks and Kurds. There are four times more Turks than Kurds. Write an equation and solve it to find out how many Kurds live in Turkey.

9-58. In 1912, Japan gave the United States several thousand flowering cherry trees as a symbol of friendship. Similarly, the nation of Cameroon plans to give flowering Satta trees to other countries this year. When asked how to decide which Satta trees make good gifts, Cameroon's chief arborist explained:

"We plant Satta trees when they are 6 cm tall, and they grow 9 cm every year. The trees only flower when they are taller than 150 cm."

It is very important that the trees Cameroon gives flower this year! It would be considered an insult to receive a tree that did not bloom. Luckily, Cameroon has many groves of Satta trees from which to select its gifts. How old must the trees be so that they will flower within the year?

a. Discuss with your study team whether an inequality or an equation is appropriate for this situation. Be prepared to share your reasoning.

b. Write and solve a mathematical sentence to determine how old the trees can be so that they flower this year.

c. Later, the arborist added:

"I almost forgot to tell you! When the trees become very old, they stop flowering. Make sure you choose trees that are no more than 240 cm tall!"

Discuss with your team how you can use this additional information to make sure you choose trees that will flower. Be prepared to share your answer with the class.

Review & Preview

9-59. Solve the inequalities below for the given variables. Represent your solutions on a number line.

a. $3(2k-1)<9$

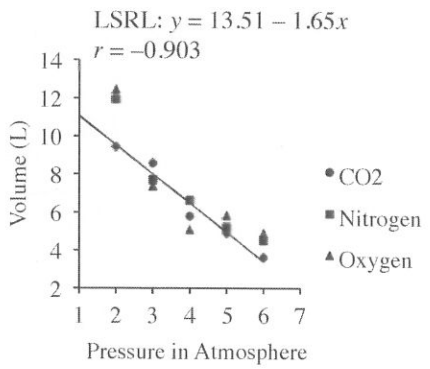
b. $\frac{2p}{5} \le 6$

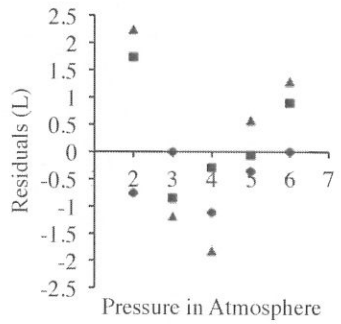
c. $-2+8n>2$

d. $7t-4>2t-4$

9-60. Brett is working in the chemistry lab trying to determine if he can predict the volume of three different gasses (nitrogen, oxygen, and carbon dioxide) based upon the pressure applied to them. He kept all other characteristics of the gasses constant. Brett performed the following analysis:

	Volume		
Pressure	Nitrogen	Oxygen	CO2
2	11.94	12.45	9.45
3	7.71	7.37	8.56
4	6.62	5.09	5.79
5	5.2	5.83	4.9
6	4.51	4.9	3.61





a. Discuss the association, including slope and R-squared.

b. What is the residual with the greatest magnitude and what point does it belong to?

c. Using the LSRL model, estimate the volume of a gas at 2.5, 4.5 and 6.5 atmospheres. Use an appropriate precision.

d. How well would this linear model work in predicting more extreme pressures? Support your answer.

9-61. Use your graphing shortcuts to graph $y=-2x+3$. Identify the x- and y-intercepts.

9-62. Find the equation of the line with slope $-\frac{3}{5}$ passing through the point $(-6, 2)$.

9-63. Solve the quadratic equation below. Check your solutions with a calculator.

$$3x^2 + 2.5x = 12.5$$

9-64. Factor the expressions below completely, if possible.

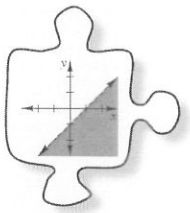
a. $4x^2 - 20x + 25$

b. $x^2 + 11x - 2$

c. $3x^2 - 12x$

d. $10x^2 - 35x - 20$

9.3.1 What if the inequality has two variables?



Graphing Linear, Two-Variable Inequalities

In Section 9.2, you learned how to use an inequality with one variable to help solve a word problem. You also discovered that a one-variable inequality can have zero, one, or more solutions and that these solutions can be represented on a number line. But what if an inequality has two variables? What is a solution to a two-variable inequality? How could these solutions be represented graphically?

9-65. EXAMINING THE SOLUTIONS OF A LINEAR EQUATION

Find your graph of $y = -2x + 3$ from problem 9-61. Compare your graph with the poster graph provided by your teacher.

a. Is the point $(-1, 5)$ a solution to the equation $y = -2x + 3$? How can you tell by looking at the graph? How can you tell by using the equation?

b. Is the point $(2, -1)$ a solution? What about the point $(0, 0)$? Justify each conclusion with both the graph and the equation.

c. What determines if a point lies on the line? What is the difference between the points on the line and the points not on the line?

9-66. GRAPHING A LINEAR, TWO-VARIABLE INEQUALITY

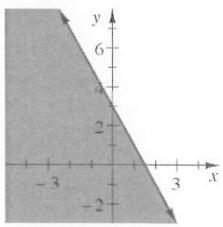
In problem 9-65, you found that the points on the line are the *only* points that make the equation $y = -2x + 3$ true. But what if you want to graph the solutions for the inequality $y \geq -2x + 3$? How will that graph differ from the graph of $y = -2x + 3$? Consider this question as you follow the steps below.

a. Your team will be given a list of points to test in the inequality $y \geq -2x + 3$. For each point that makes the inequality true, place a sticky dot on that point on the class graph.

b. Now examine the solutions shown on the graph. With your team, discuss the questions below. Be ready to share your conclusions with the class.

- Are there any points on the graph that you suspect are solutions but do not have a sticker?
- Are there any stickers that you think may be misplaced? If so, verify these points so that you can have a complete graph of the solutions.
- What about the points on the line? Are they all solutions to the inequality $y \geq -2x + 3$? Why or why not?
- How many solutions are there?
- Why aren't any of the solutions located below the line?

9-67. What else can you learn about solutions of linear inequalities? Think about this as you answer the questions below with your team.



a. What if the graph were shaded like the one at right? What inequality would correspond with this graph?

b. Heidi asks, "What if I changed the inequality to be $y < -2x + 3$? Now what would the graph look like?" Discuss this with your teammates and decide the best way to represent the solutions to the inequality $y < -2x + 3$. Be prepared to share your graph with the class.

9-68. Graph the inequalities below on graph paper. For each inequality:

- Graph the boundary as either a solid or a dashed line.
- Shade the region that makes the inequality true.

a. $y > -\frac{1}{3}x - 1$

b. $y \leq 4x + 2$

c. $y < \frac{5}{2}x + 3$

d. $2x - y \leq 5$

9-69. In your Learning Log, explain how to graph a linear inequality. Be sure to address the questions below. Title this entry "Graphing Linear Inequalities" and include today's date.

- How can you determine if the line is part of the solution?
- How can you determine which side of the line the solution belongs to?
- What point(s) is (are) easiest to test?
- How many points do you need to test?

MATH NOTES

METHODS AND MEANINGS

Curve Fitting an Exponential Function

You can find an exponential function that goes through two points (if both points are above the x-axis). Recall that an exponential function with an asymptote of the x-axis has an equation of the form $y = ab^x$.

To find an exponential function that goes through two given points, create a system of equations by substituting one (x, y) point into $y = ab^x$, then substituting the other point. Rewrite both equations in "$a =$" form. Solve the system with the equal values method to find a and b and now you can write the equation.

For example, find an exponential function that passes through (2, 14) and (5, 112). Create a system of equations by substituting $(x, y) = (2, 14)$ into $y = ab^x$, and then substituting again with $(x, y) = (5, 112)$:

$14 = ab^2$ and $112 = ab^5$ Rewrite as $a = \frac{14}{b^2}$ and $a = \frac{112}{b^5}$.

Use the equal values method to find b:

$$\frac{14}{b^2} = \frac{112}{b^5}$$
$$14b^5 = 112b^2$$
$$14 \cdot \frac{b^5}{b^2} = 112$$
$$14b^3 = 112$$
$$b^3 = \frac{112}{14} = 8$$
$$b = 2$$

Use either original equation to find a:

$$14 = ab^2$$
$$14 = a(2)^2$$
$$\frac{14}{4} = a$$
$$3.5 = a$$

The equation of the line that passes through the two given points is $y = 3.5 \cdot 2^x$.

Review & Preview

9-70. Represent the solutions to the inequalities below on a number line.

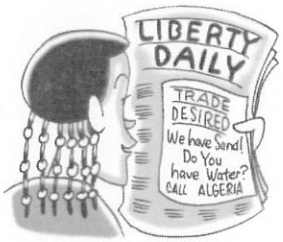
a. $3x - 2 < 10$

b. $5x - 1 - 3x \geq 4x + 5$

c. $2(x + 2) > 10 - x$

d. $4(x - 3) + 5 \geq -7$

9-71. Algeria has decided to take out an advertisement in the U.N newspaper, *Liberty Daily*. The newspaper charges a base fee of \$1200 for an ad. There is an additional fee of \$300 for every inch in height. If Algeria is willing to spend any amount up to (and including) \$2700, what choices does the country have for the height of the ad?



9-72. If the graph of an exponential function passes through the points $(1,6)$ and $(4,48)$, find an equation of the function. Refer to the Math Notes box in this lesson if you need help.

9-73. Line m has intercepts $(-7,0)$ and $(0,-2)$.

a. Find the equation of line m.

b. Is the point $(49,-16)$ also on line m? How do you know?

9-74. Thui made the following hypotheses: $2n-1<5$ and $n+1\le 2n$. Which of the following conclusions can she make?

a. $n\ge 1$ and $n\le 3$

b. $n\ge 1$ and $n<3$

c. $n>1$ and $n\le 3$

d. $n>1$ and $n<3$

9-75. **Multiple Choice:** Which of the expressions below is a factor of $6m^2+7m-5$?

a. $2m+1$ b. $m+5$ c. $2m-5$ d. $3m+5$

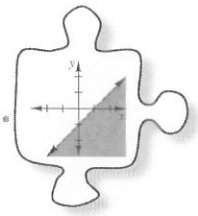
9-76. For the quadratic function $f(x)=x^2+6x+11$:

a. Use the idea of completing the square to write it in graphing form.

b. State the vertex and sketch a graph of the parabola.

c. Use the graph to explain why the equation $x^2+6x+11=0$ has no real solutions.

9.3.2 What if the inequality is not linear?



Graphing Linear and Non-Linear Inequalities

In Lesson 9.3.1, you discovered that the solutions of a linear inequality with two variables can be represented by a shaded region on one side of the line. But how can the graph of an inequality help solve a problem? And what happens when the inequality is not linear? Consider these questions as you complete the following problems with your study team.

9-77. Review what you learned about graphing inequalities in Lesson 9.3.1 by graphing the inequality below on graph paper.

$$y \geq -\tfrac{5}{3}x - 3$$

a. What is the minimum number of points you need to test in order to know which side of the line the solution falls on?

b. Orville thinks that using the point (0, 0) to test this inequality is a great idea. Why is using this point so convenient?

c. Anita decided to use the point (–3, 2) to test the inequality. Test the inequality with her point. Does this point help her decide which side to shade? Why or why not?

9-78. FOREIGN AID

One of the purposes of the United Nations is to have nations work together to help each other. Recently, the members of the U. N. decided to give grants to poor countries to help reduce poverty. However, the United Nations only has the resources to help those countries with the greatest need. Therefore, it was decided that only countries in which the number of people in poverty is *more than* one-half of its total population would receive foreign aid.

a. A **constraint** is a limitation or a restriction. Write an inequality that represents the **constraints** on receiving foreign aid. Let x represent the population and y represent the number of people in poverty.

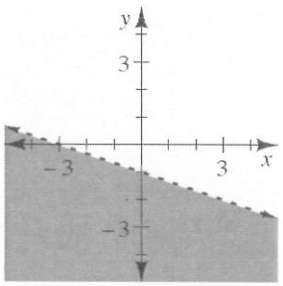
b. On the Lesson 9.3.2 Resource Page provided by your teacher (also available at www.cpm.org), find the graph that shows the number of people in poverty per the population for each of the countries being considered for foreign aid. Carefully graph your inequality from part (a) on this data graph. Which countries should receive foreign aid?

9-79. What if an inequality is non-linear? Decide with your team how to graph the inequality $y < x^2 - 4x + 3$ on graph paper. Your graphing shortcuts can help.

9-80. With your team, use graphing shortcuts to graph the following inequalities on graph paper. Use an xy-coordinate grid for all three parts.

a. $y < 4(1.15)^x$ b. $y \geq \frac{1}{4}(x-6)^2 + \frac{3}{4}$ c. $x < 2$

9-81. Write the inequality for the solution graphed at right. Be prepared to explain how you found your rule.



9-82. Graph the inequality $y < |2x - 1|$. Be ready to share your graph with the class.

MATH NOTES

METHODS AND MEANINGS

Solving One-Variable Linear Inequalities

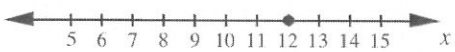
To solve a one-variable linear inequality, first treat the problem as if it were an equality. The solution to the equality is called the **boundary point**. For example, $x = 12$ is the boundary point for the inequality $10 - 2(x-3) \geq -8$, as shown below.

Problem: $10 - 2(x-3) \geq -8$

First change the problem to an equality and solve for x:

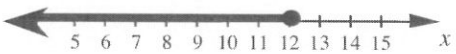
$$10 - 2(x-3) = -8$$
$$10 - 2x + 6 = -8$$
$$-2x + 16 = -8$$
$$-2x = -24$$
$$x = 12$$

Since the original inequality is true when $x = 12$, place your boundary point on the number line as a solid point. Then test one value on either side in the *original* inequality to determine which set of numbers makes the inequality true.



Test: $x = 8$	**Test:** $x = 15$
$10 - 2(8-3) \geq -8$	$10 - 2(15-3) \geq -8$
$10 - 2(5) \geq -8$	$10 - 2(12) \geq -8$
$0 \geq -8$	$-14 \geq 17$
TRUE!	FALSE!

Therefore, the solution is $x \leq 12$.



When the inequality is $<$ or $>$, the boundary point is *not* included in the answer. On a number line, this is indicated with an open circle at the boundary point.

Review & Preview

9-83. **Multiple Choice:** Which of the expressions below is a factor of $6x^2 + 7x - 20$?

a. $3x - 4$ b. $2x - 5$ c. $3x + 4$ d. $4x - 3$

9-84. Graph the inequalities below on graph paper.

a. $y \le -x + 5$ b. $y > \frac{2}{3}x - 1$

9-85. Write and solve an inequality for this situation.

To honor 50 years in business, All Strikes Bowling is having an anniversary special. Shoes rent for \$1.25 and each game is \$0.75. If Charlie has \$20 and needs to rent shoes, how many games can he bowl?

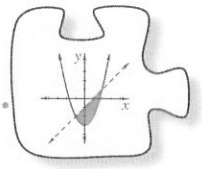
9-86. Solve the following equations and inequalities for x. Check your solution(s), if possible.

a. $\frac{3}{x} = 9$ b. $\sqrt{x} = 4$ c. $x^2 = 25$ d. $2(x - 3) > 4$

9-87. During a race, Bernie ran 9 meters every 4 seconds, while Wendel ran 2 meters every second and got a 9-meter head start. If the race was 70 meters long, did Bernie ever catch up with Wendel? If so, when? Justify your answer.

9-88. Determine the number of times the graph of $y = 5x^2 + 7x - 6$ intersects the x-axis using *two different methods*. The answers from each method should match.

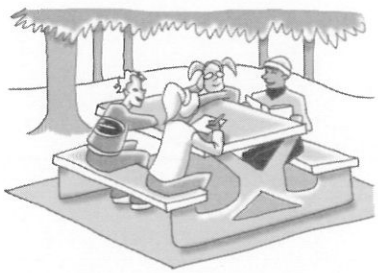
9.4.1 How can I represent it?



Systems of Inequalities

In Chapter 4 you learned that the solution to a system of equations is a point that makes both equations true. But what about the solution of a system of two inequalities? How can you represent these solutions on a graph? How many solutions can a system of inequalities have?

Consider these questions as you learn how to graph a system of inequalities.



9-89. Find your graphs for problem 9-84.

a. Compare your solution graphs for $y \le -x+5$ and $y > \frac{2}{3}x-1$ with those of your teammates. Correct any errors. Be sure to focus on whether the boundary line should be included in each graph.

b. What would the graph of the system of inequalities look like? Consider the system of inequalities below. Which points are solutions to this system (that is, which points make *both* inequalities true)?

$$y \le -x+5$$
$$y > \frac{2}{3}x-1$$

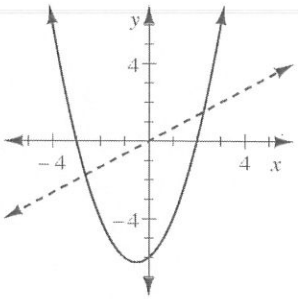
c. If you have not done so already, verify your solution region from part (b) algebraically by substituting the coordinates of a point from your solution region into each inequality.

d. How can you be sure that this region is the only set of points that makes both inequalities true?

9-90. Draw a graph of the region satisfying both constraints below. Start by graphing the boundary lines and then test points to find the region that makes both inequalities true.

$$y < x+2$$
$$y \le 10-\frac{3}{4}x$$

9-91. HOW MANY REGIONS?



When graphing the system of inequalities below, Reyna started with the boundary graph of each constraint shown at right.

$$y \le x^2 + x - 6$$
$$y > \tfrac{2}{3}x$$

a. Why is the line dashed while the parabola is not?

b. Find a copy of Reyna's graph on the Lesson 9.4.1B Resource Page provided by your teacher. How many possible solution regions are there? Carefully count each region with your teammates.

c. Pick a point in each region and test it in the system of inequalities. Shade any regions that contain solutions to both inequalities. How many regions make up the solution to this system?

d. Why is (0, 0) not a good point to use to test for this solution?

9-92. How does changing the inequality affect the solution graph? Notice that each system of inequalities below uses the same boundary graphs as Reyna's graph from problem 9-91. However, notice that this time the constraints are slightly altered.

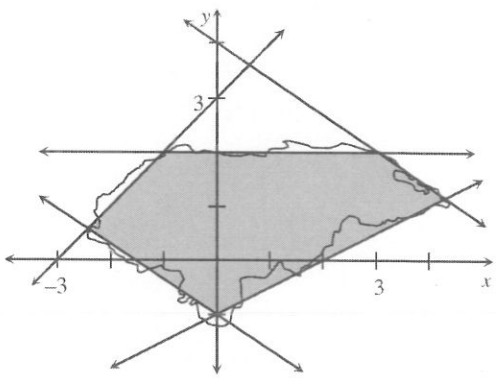
With your teammates, devise a method to determine which region (or regions) are solutions for each system. Shade the appropriate regions on your resource page.

a. $y \ge x^2 + x - 6$, $y > \tfrac{2}{3}x$

b. $y \ge x^2 + x - 6$, $y < \tfrac{2}{3}x$

c. $y \le x^2 + x - 6$, $y < \tfrac{2}{3}x$

9-93. The United Nations asked every nation to write constraints that best approximate its country's shape (the U.N. thinks this will help find each country's area). Honduras sent in its inequalities, but some of the information is unreadable. With your study team, determine the missing parts of the inequalities and rewrite them on your paper.



y [illegible] $x + 3$ $\quad$ $y \ge \tfrac{1}{2}x -$ [illegible]

[illegible] 2 $\quad$ y [illegible] $-\tfrac{2}{3}x + 4$

y [illegible] $-\tfrac{2}{3}x - 1$

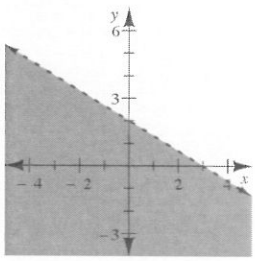
MATH NOTES

METHODS AND MEANINGS

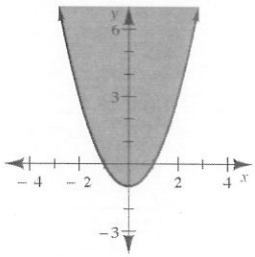
Solving Inequalities with Two Variables

To graph solve an inequality with two variables, first graph the boundary line or curve. If the inequality does not include an equality (that is, if it is $>$ or $<$ rather than $\geq$ or $\leq$), then the graph of the boundary is dashed to indicate that it is not included in the solution. Otherwise, the boundary is a solid line or curve.

Once the boundary is graphed, choose a point that does not lie on the boundary to test in the inequality. If that point makes the inequality true, then the entire region in which that point lies is a solution. Examine the two examples below. There are infinite solutions to each of the inequalities. The shaded portion of the graph is a diagram of all of the solutions.



$y < -\frac{2}{3}x + 2$



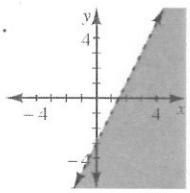
$y \geq x^2 - 1$

Review & Preview

9-94. Match each graph below with the correct inequality.

a. $y > -x + 2$ b. $y < 2x - 3$ c. $y \geq \frac{1}{2}x$ d. $y \leq -\frac{2}{3}x + 2$

1.



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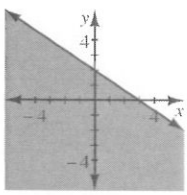
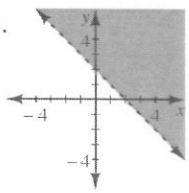


3.



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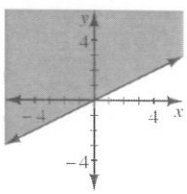


9-95. Solve each inequality below. Represent the solutions on number lines.

a. $7x-2<3+2x$

b. $\frac{1}{3}x \geq 2$

c. $3(2m-1)-5m \leq -1$

d. $2k+3 \leq 2k+1$

9-96. Three years ago the average price of a movie ticket was \$8.75 and now it is \$11.00. What was the annual multiplier and the percent increase?

9-97. Factor the following quadratics completely.

a. $5x^3+13x^2-6x$

b. $6t^2-26t+8$

c. $6x^2-24$

9-98. When a family with two adults and three children bought tickets for a movie, they paid a total of \$27.75. The next family in line, with two children and three adults, paid \$32.25 for the same movie. Find the adult and child ticket prices by writing a system of equations with two variables.

9-99. Solve the equation below by completing the square. Give you answer in exact (radical) form.

$$x^2-6x+3=0$$

9-100. **Multiple Choice:** Which of the points below is a solution of $y<|x-3|$?

a. $(2, 1)$

b. $(-4, 5)$

c. $(-2, 8)$

d. $(0, 3)$

9.4.2 How can I apply it?

More Systems of Inequalities

9-101. Review what you learned about systems of inequalities in Lesson 9.4.1 by graphing the system of inequalities at right on graph paper. Carefully shade the region of points that make *both* inequalities true.

$$y \le |x| + 4$$
$$-x + 4y \ge 4$$

9-102. SEARCH AND RESCUE

"I'm completely lost... water everywhere I can see... both burners have failed... Wait! I see land. I'm going to try to land. I think it's..."

Those were the last words heard from Harold in his hot-air balloon. The last time the balloon showed up on radar, it was near the Solomon Islands in the Pacific Ocean.

Your Task: Your team must determine where to send the search-and-rescue teams! Use the following reports along with the map on the Lesson 9.4.2 Resource Page and look carefully for information that will help you draw constraints on where the balloon might be found. Give the search constraints to the search-and-rescue team as a system of inequalities. Be sure to identify the probable landing site on the map.

Basic facts of the case:

The balloon departed from the airport at the very northern tip of the Philippines. The flight was supposed to follow a straight path *directly* to an airport in French Polynesia.

The balloon's last known location was at (–1000, 1000) near the Solomon Islands.

Pilot's report from a nearby airplane:

"We were on our way from Australia, when we saw a hot-air balloon sinking rapidly. I am certain that it crashed south of our flight path. When we left Australia, we traveled 2000 km north for every 3000 km east that we flew."

Phone call received today:

"I was a passenger on a flight that flew directly from French Polynesia to Indonesia. I was looking out my window when I saw the balloon going down to the north of where we were flying."

9-103. Notice that each system of inequalities below contains the same boundary lines. On graph paper, graph the boundaries for the system on one set of axes. Then, for each pair of inequalities, work with your teammates to decide which region is the solution, if a solution exists. Be ready to share your conclusions with the class.

a. $y \le \frac{2}{3}x + 3$
$y \ge \frac{2}{3}x$

b. $y \le \frac{2}{3}x + 3$
$y \le \frac{2}{3}x$

c. $y \ge \frac{2}{3}x + 3$
$y \le \frac{2}{3}x$

9-104. In your Learning Log, describe your method for graphing systems of inequalities for a student who has missed class for the last couple of days. Be sure to include examples and important details. Title this entry "Graphing Systems of Inequalities" and include today's date.

Review & Preview

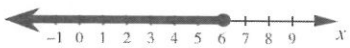
9-105. Graph and shade the solution for the inequality below.

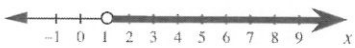
$$y \le x^2 + 2x - 8$$

9-106. Graph and shade the solution for the system of inequalities below.

$$y \ge \frac{3}{4}x - 2$$
$$y < -\frac{1}{2}x + 3$$

9-107. Write the inequality that represents the x-values highlighted on each number line below.

a. 

b. 

c.

d.

9-108. Determine if the following statements are true or false.

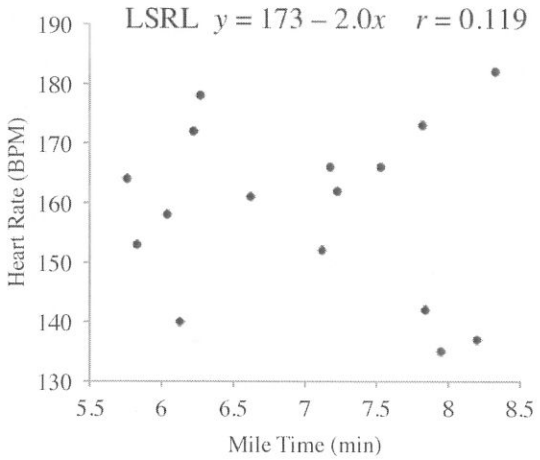
a. $|-6| < 4$

b. $|-3+5| > 2.5$

c. $4 \ge |0|$

d. $|-4+3| > 1$

9-109. Coach Ron has 15 athletes trying out for 12 openings on the varsity basketball team. As part of the tryout he has each player run a mile and Ron records their time and heart rate as they finish. Coach Ron made a scatterplot as follows.

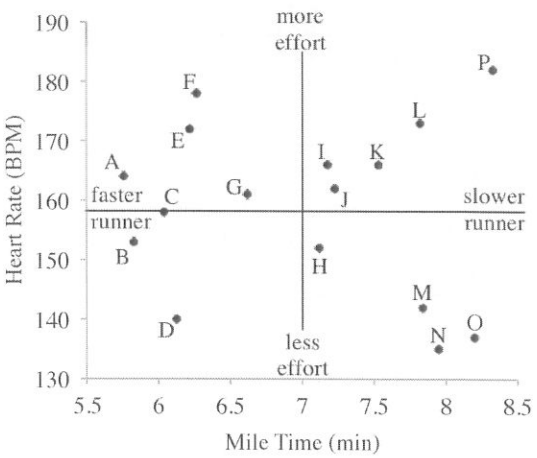


	Min.	BPM
A	5.76	164
B	5.83	153
C	6.04	158
D	6.13	140
E	6.22	172
F	6.27	178
G	6.62	161
H	7.12	152
I	7.18	166
J	7.23	162
K	7.53	166
L	7.82	173
M	7.84	142
N	7.95	135
O	8.2	137
P	8.33	182

a. Describe the association to Coach Ron in detail.

Ron's assistant Cheryl believes they can still find something useful in the results. She proposes that if heart rates could be considered the level of effort given, some athletes may stand out as more or less desirable for the team.

Cheryl divided the scatterplot into four quadrants and put the new labels on the axes. She also labeled each point. She drew the following scatterplot.



b. Coach Cheryl believes that the farther a player is from the center, the more they help or hurt the team. Give specific advice to Coach Ron about players D, F, N, O and P.

9-110. For the Spring Festival, the Math Club is selling rulers for \$1 and compasses for \$2.50.

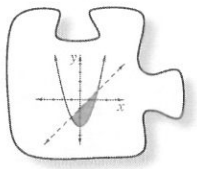
a. While the club would like to sell as many items as they can to raise funds, they need to make at least \$15.00 to break even. Write an inequality to represent this situation. Let r = the number of rulers sold and c = the number of compasses sold.

b. School rules state that the club can sell a maximum of 25 items for the festival. Write an inequality for this constraint (limitation).

c. Graph the inequalities from parts (a) and (b) on the same set of axes so that compasses are represented on the x-axis and rulers are represented on the y-axis. Find the region of points that are solutions to each of them. Can this region fall below the x-axis or to the left of the y-axis? Why or why not?

d. What do the points in the solution region represent?

9.4.3 How can I use inequalities to solve problems?



Applying Inequalities to Solve Problems

Today you will pull together all of the mathematics you have studied in this chapter and apply it to solve an application problem.

9-111. UNITED NATIONS TO THE RESCUE

As a representative of your country, you have been sent the following letter and given an important task:

> *Dear Representative to the United Nations:*
>
> *A critical matter has come to the attention of the United Nations. In the past, when a catastrophe struck a part of the world, the U. N. gathered supplies to give to people in need. Unfortunately, because the U. N. had to collect supplies from each country at the time of the catastrophe, it was always quite a few days before the supplies could be sent to the areas that needed them the most.*
>
> *A recommendation has come before the U. N. to create a supply of food and medicine packages for future emergencies. Each food package will be able to feed several hundred people, while each medicine package will supply one first-aid station. I am asking each country to donate the same number of packages so each country shares the burden equally.*
>
> *I am asking each country to determine how many food and medicine packages they are able to give. You will present your findings at today's United Nations meeting. Please be certain to use the information that your country's Budget Committee has prepared to help you decide how many packages you can afford.*
>
> *Best of luck, and may our efforts make our world a better place!*
>
> *Sincerely,*
> *The Secretary General of the United Nations*

After consulting with your country's Budget Committee, your teacher will supply you with some information that will help decide how many food and medicine packets your country can afford.

Problem continues on next page. →

9-111. *Problem continued from previous page.*

Your Task: To communicate your country's budget constraints, write an inequality expressing how many food and medicine packages your country is able to give. Let x equal the number of food packages and y equal the number of medicine packages.

On the Lesson 9.4.3B Resource Page provided by your teacher, graph the solution region representing the number of medicine and food packets that can be donated by your country. Be prepared to share your graph with the other countries of the United Nations.

9-112. As a member of the United Nations, you must consider each of the following proposals. In each case, assume that the United Nations would like to receive as many emergency supplies as possible, while still having each nation give equally.

a. One proposal is that each country gives 185 medicine packages. How many food packages should the United Nations require from each country in this case? Explain how you made your decision.

b. Another proposal is to get the largest number of medicine packages possible. What is the largest number of medicine packages that each country can offer? How did you find your answer?

9-113. EXTENSION

A last-minute proposal suggests balancing the number of food and medicine supplies. For instance, if a country gives 150 food packages, then they would also give 150 medicine packages. How many food and medicine packages should the United Nations require from each country in this case? Explain how you determined your solution.

Review & Preview

9-114. While setting up a mathematical sentence to solve a problem, Paulina and Aliya came up with the equations below. Since the equations did not look alike, the girls turned to you for help.

$$\text{Paulina: } 4x+2y=6$$

$$\text{Aliya: } 12x+6y=18$$

a. Are these equations equivalent? That is, will the graph of each line be the same? Explain how you know.

b. Find another equation that is equivalent to both of these. How did you find your equation?

9-115. The town you live in has decided to limit the amount of trash thrown out each month. Your town, which has 3280 homes, has asked each household to keep track of how many pounds of trash they produce during a month. In addition, the town council has found that other sources of trash, such as local businesses, combine to create 1500 lbs of trash each month. If the town has a goal of creating *less than* 50,000 lbs of trash, how much trash should a household be limited to? Write an inequality for this situation and solve it.

9-116. Solve the following inequalities for the given variable and represent the solutions on a number line.

a. $2<2m-8$

b. $\frac{1}{3}x-1\le -3$

c. $5(2x-8)+24>3(4+2x)$

d. $5+2k<k-2+k$

9-117. Solve the quadratic equation below *twice*, once using the Quadratic Formula and once by completing the square. Which was easier?

$$x^2-10x+21=-4$$

9-118. Read the following problem. Then decide which system of equations below can represent this situation.

Multiple Choice: The length of a rectangle is 4 units longer than twice its width. If the area is 126 square units, find the length and width.

a. $w = 2l + 4$, $wl = 126$

b. $l = 2w + 4$, $l + w = 126$

c. $w = 2l + 4$, $l + w = 126$

d. $l = 2w + 4$, $wl = 126$

9-119. This problem is a checkpoint for writing exponential equations. It will be referred to as Checkpoint 9.

Write an exponential equation to represent the situation and answer the question.

a. If the cost of a loaf of bread is now \$2.75 and is increasing at 5% per year, what will it cost 10 years from now?

b. The population of Flood River City is now 42,000. Experts predict the population will decrease 25% each year for the next five years. What will be the population in five years?

c. A share of Orange stock that was worth \$25 in 2000 was worth \$60 in 2010. What is the multiplier and percent increase?

Check your answers by referring to the Checkpoint 9 materials located at the back of your book.

If you needed help solving these problems correctly, then you need more practice. Review the Checkpoint 9 materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to do like these quickly and easily.

Chapter 9 Closure What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect about what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for connections between ideas as well as connections with material you learned previously.

① TEAM BRAINSTORM

What have you studied in this chapter? What ideas were important in what you learned? With your team, brainstorm a list. Be as detailed as you can. To help get you started, a list of Learning Log entries and Math Notes boxes are below.

What topics, ideas, and words that you learned *before* this chapter are connected to the new ideas in this chapter? Again, be as detailed as you can.

How long can you make your list? Challenge yourselves. Be prepared to share your team's ideas with the class.

Learning Log Entries

- Lesson 9.1.2 – Quadratic Formula
- Lesson 9.1.4 – Choosing a Strategy to Solve Quadratics
- Lesson 9.3.1 – Graphing Linear Inequalities
- Lesson 9.4.2 – Graphing Systems of Inequalities

Math Notes

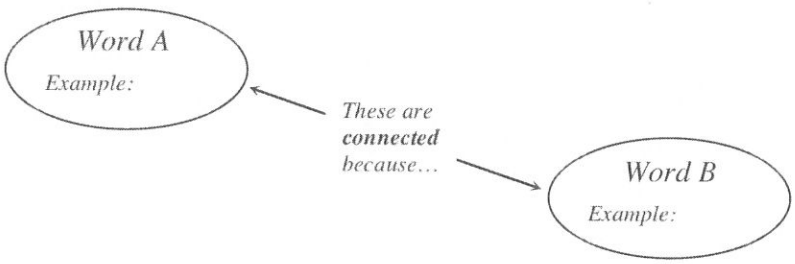
- Lesson 9.1.1 – Zeros and Roots of Quadratics
- Lesson 9.1.2 – The Quadratic Formula
- Lesson 9.1.3 – Solving a Quadratic Equation
- Lesson 9.1.4 – Simplifying Square Roots
- Lesson 9.2.1 – Inequality Symbols
- Lesson 9.3.1 – Curve Fitting an Exponential Function
- Lesson 9.3.2 – Solving One-Variable Linear Inequalities
- Lesson 9.4.1 – Solving Inequalities with Two Variables

② MAKING CONNECTIONS

Below is a list of the vocabulary used in this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

Zero Product Property	boundary	coordinates
quadratic equation	graph	inequality
complete the square	region	Quadratic Formula
system of inequalities	standard form	factoring
number line	solution	

Make a concept map showing all of the connections you can find among the key words and ideas listed above. To show a connection between two words, draw a line between them and explain the connection, as shown in the model below. A word can be connected to any other word as long as you can justify the connection. For each key word or idea, provide an example or sketch that shows the idea.



Your teacher may provide you with vocabulary cards to help you get started. If you use the cards to plan your concept map, be sure either to re-draw your concept map on your paper or to glue the vocabulary cards to a poster with all of the connections explained for others to see and understand.

While you are making your map, your team may think of related words or ideas that are not listed here. Be sure to include these ideas on your concept map.

③ PORTFOLIO: EVIDENCE OF MATHEMATICAL PROFICIENCY

Carefully copy your work from problem 9-102, modifying and expanding it if necessary, to showcase your understanding of solving inequalities.

Solve problem 9-37, Moe's Yo, in your portfolio to show your growth in understanding parabolas, including showcasing your new ability to solve quadratic equations.

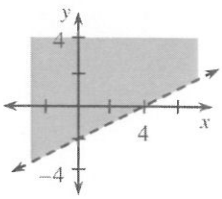
Your teacher may give you the Chapter 9 Closure Resource Page: Quadratic Multiple Representations Graphic Organizer. Complete the Resource Page for Moe's Yo from problem 9-37. Showcase your current understanding of the multiple representations of a quadratic function.

④ WHAT HAVE I LEARNED?

Most of the problems in this section represent typical problems found in this chapter. They serve as a gauge for you. You can use them to determine which types of problems you can do well and which types of problems require further study and practice. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you still need to work on.

Solve each problem as completely as you can. The table at the end of the closure section has answers to these problems. It also tells you where you can find additional help and practice with problems like these.

CL 9-120. Write an inequality that represents the graph at right.



CL 9-121. Is the point $(0, 4)$ a solution to the system of inequalities at right? Justify your answer.

$$y \le -3x + 4$$
$$y > x^2 + 3x - 2$$

CL 9-122. Factor these quadratic expressions completely, if possible.

a. $x^2 + x - 30$

b. $-3x^3 + 23x^2 - 14x$

c. $2x^2 - 5x + 4$

d. $6x^3 + 10x^2 - 24x$

CL 9-123. Solve each inequality below for the given variable. Then represent each solution on a number line.

a. $4x - 3 \geq 9$

b. $3(t + 4) < 5$

c. $\frac{2y}{7} < 8$

d. $5x + 4 > -3(x - 8)$

CL 9-124. Brian was holding a ballroom dance. He wanted to make sure girls would come, so he charged boys \$5 to get in but girls only \$3. The 45 people who came paid a total of \$175. How many girls came to the dance?

CL 9-125. Solve each quadratic equation using the specified method.

a. The Quadratic Formula

$0 = 3x^2 + 4x - 7$

b. Factoring

$x^2 - 3x - 18 = 0$

c. Completing the square

$x^2 + 4x + 1 = 0$

d. Using a graph

$2x^2 + 5x - 12 = 0$

CL 9-126. Given the quadratic function $f(x) = (x - 1)^2 - 4$:

a. State the location of the vertex.

b. Determine the x-intercepts.

c. Sketch a graph of the function.

CL 9-127. Graph the system of inequalities below on graph paper.

$$y < x^2$$
$$y \geq x + 2$$

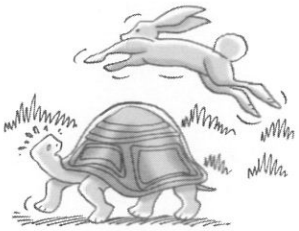
CL 9-128. Lew says to his granddaughter Audrey, "Even if you tripled your age and added 9, you still wouldn't be as old as I am." Lew is 60 years old. Write and solve an inequality to determine the possible ages Audrey could be.

CL 9-129. The cost to rent a DVD has decreased 10% per year over the past several years.

a. If the current cost is \$5, write an exponential equation describing this situation.

b. According to the equation, what did it cost to rent a DVD 5 years ago?

CL 9-130. The hare leaps 500 centimeters every 20 seconds. The tortoise crawls 250 centimeters every 50 seconds, but gets a 1000-centimeter head start. Use any method you know to determine how long it takes the hare to catch up to the tortoise.

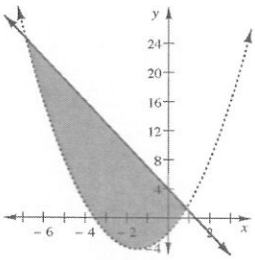
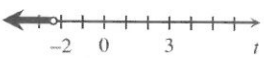
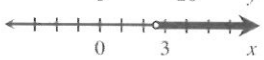


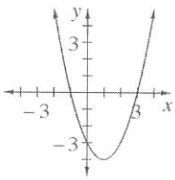
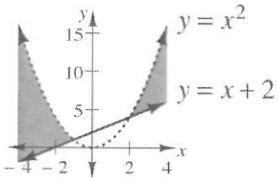
CL 9-131. Find the equation of an exponential function of the form $y = ab^x$ that passes through the points $(3, 13.5)$ and $(5, 30.375)$.

CL 9-132. Check your answers using the table at the end of the closure section. Which problems do you feel confident about? Which problems were hard? Use the table to make a list of topics you need help on and a list of topics you need to practice more.

Answers and Support for Closure Activity #4
What Have I Learned?

Note: MN = Math Note, LL = Learning Log

Problem	Solution	Need Help?	More Practice
CL 9-120.	$y > \frac{1}{2}x - 2$	Section 9.3 MN: 9.4.1 LL: 9.3.1	Problems 9-81, 9-84, 9-94, and 9-105
CL 9-121.	Yes, because it makes both inequalities true; the point (0, 4) lies on the solution graph $x^2 + 3x - 2 < y \le -3x + 4$. Therefore, it is a solution to the system of inequalities. 	Section 9.4 LL: 9.4.2	Problems 9-31, 9-100, and 9-106
CL 9-122.	a. $(x+6)(x-5)$ b. $x(-3x+2)(x-7)$ c. not factorable d. $2x(x+3)(3x-4)$	Section 8.1 MN: 8.1.4 LL: 8.1.3 and 8.1.5	Problems CL 8-112, 8-74, 9-33, 9-64, and 9-97
CL 9-123.	a. $x \ge 3$ b. $t < -\frac{7}{3}$  c. $y < 28$ d. $x > 2.5$ 	Section 9.2 MN: 9.2.1 and 9.3.2	Problems 9-51, 9-59, 9-70, 9-95, 9-107, and 9-116
CL 9-124.	Let b = # boys, g = # girls. $5b + 3g = 175$, $b + g = 45$. 25 girls came to the dance.	Chapter 4 Checkpoints 7A and 7B MN: 4.2.3 LL: 4.2.3	Problems CL 7-118, CL 8-118, 9-30, 9-38, 9-50, and 9-98

Problem	Solution	Need Help?	More Practice
CL 9-125.	a. $x = 1$ or $x = -\frac{7}{3}$ b. $x = 6$ or $x = -3$ c. $x = -2 \pm \sqrt{3} \approx -0.27, -3.73$ d. $x \approx -4$, or 1.5	Section 9.1 MN: 9.1.1, 9.1.2, and 9.1.3 LL: 9.1.2 and 9.1.4	Problems 9-6, 9-17, 9-18, 9-27, 9-28, 9-39, 9-52, 9-55, 9-63, 9-99, and 9-117
CL 9-126.	a. $(1, -4)$ b. $x = 3, -1$ c. See graph at right. 	Lessons 8.2.3 and 8.2.5 MN: 8.2.4	Problems 9-21, 9-76, and 9-88
CL 9-127.		Sections 9.3 and 9.4 MN: 9.4.1 LL: 9.3.1 and 9.4.2	Problems 9-106 and 9-110
CL 9-128.	$3A + 9 < 60$ $A < 17$ Audrey is less than 17 years old.	Section 9.2	Problems 9-71, 9-85, 9-110, and 9-115
CL 9-129.	a. $f(x) = 5(0.9)^x$ b. $5(0.9)^{-5} \approx \$8.47$	Checkpoint 9	Problems 8-20, 9-22, 9-43, 9-96, and 9-119
CL 9-130.	The hare catches up to the tortoise after 50 seconds.	Lessons 2.2.2 and 2.2.3	Problems 7-27, 8-41, 9-32, and 9-87
CL 9-131.	$y = 4(1.5)^x$	Lessons 7.2.1 and 7.2.2 MN: 9.3.1	Problems CL 7-112 and 9-72